NON LINEAR BUCKLING ANALYSIS OF LAMINATED COMPOSITE TWISTED PLATES

A THESIS SUBMITTED IN PARTIAL FULFILMENT

OF THE REQUIREMENTS FOR THE DEGREE OF

Master of Technology

in

Structural Engineering

Submitted By

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MAY 2014

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DEPARTMENT OF CIVIL ENGINEERING

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CERTIFICATE

This is to certify that the thesis entitled "**NON LINEAR BUCKLING ANALYSIS OF LAMINATED COMPOSITE TWISTED PLATES**", submitted by **S Vara Prasanth** bearing **Roll no. 212CE2045** in partial fulfillment of the requirements for the award of ***Master of Technology*** in the Department of Civil Engineering, National Institute of Technology, Rourkela is an authentic work carried out by him under my supervision and guidance.

To the best of my knowledge, the matter embodied in the thesis has not been submitted to any other university/institute for the award of any Degree or Diploma.

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A C K N O W L E D G E M E N T

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ABSTRACT

The twisted plate has various applications in turbine blades, compressor blades, fan blades and particularly in gas turbines. Many of these plates are subjected to in-plane load due to fluid or aerodynamic pressures. Buckling of such plates is of special importance especially if the plates are thin. Hence it is necessary to study their behaviour under different types of loads. Laminated composite materials are increasingly used as load bearing structural components in aerospace and naval structures, automobiles, pressure vessels, turbine blades and many other engineering applications because of their high specific strength and stiffness.

For a complete buckling study, a geometrically nonlinear analysis should be carried out. In a geometrically nonlinear analysis, the stiffness matrix of the structure is updated between loading increments to take into account deformations which affect the structural behaviour unlike a linear buckling analysis where the stiffness matrix is constant through the analysis.

The analysis is carried out using ANSYS software. In ANSYS, the shell 281 element with six degrees of freedom per node is used. A twelve by twelve mesh is found to give good accuracy.The buckling of twisted plates was investigated by a nonlinear analysis. The effect of number of layers, changing angle of twist, width to thickness ratio, aspect ratio, etc is also studied. It was observed in all cases that the buckling load by nonlinear analysis is lesser than that predicted by a linear analysis which proves the importance of the present study.

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Nomenclature

The principal symbols used in this thesis are presented for reference. Every symbol is used for different meanings depending on the context and defined in the text as they occur.

a, b	dimensions of the twisted panel
a/ b	aspect ratio of the twisted panel
A_{ij}, B_{ij}, D_{ij} and S_{ij}	extensional, bending-stretching coupling, bending and transverse shear stiffnesses
b/ h	width to thickness ratio of the twisted panel
dx, dy	element length in x and y-direction
dV	volume of the element
E_{11}, E_{22}	modulii of elasticity in longitudinal and transverse directions
G_{12}, G_{13}, G_{23}	shear modulii
h	thickness of the plate
k	shear correction factor
k_x, k_y, k_{xy}	bending strains
Mx, My, Mxy	moment resultants of the twisted panel
n	number of layers of the laminated panel
[N]	shape function matrix
N_{cr}	critical load
N_x, N_y, N_{xy}	in-plane stress resultants of the twisted panel
N_x^0, N_y^0, N_{xy}^0	external loading in the X and Y directions respectively
[P]	mass density parameters
q	vector of degrees of freedom
Q_x , Q_y	shearing forces
R_x, R_y, R_{xy}	radii of curvature of shell in x and y directions and radius of twist

u, v, w	displacement components in the x, y, z directions at any point
u_o, v_o, w_o	displacement components in the x, y, z directions at the midsurface
w	out of plane displacement
x_i, y_i	cartesian nodal coordinates
γ	shear strains
$\varepsilon_x\ \varepsilon_y\ \gamma_{xy}$	strains at a point
$\varepsilon_{xnl}, \varepsilon_{ynl}, \varepsilon_{xynl}$	non-linear strain components
θ_x, θ_y	rotations of the midsurface normal about the x- and y- axes respectively
λ	non-dimensional buckling load
ν	Poisson's ratio
$(\rho)_k$	mass density of kth layer from mid-plane
P	mass density of the material
$\sigma_x\ \sigma_y\ \tau_{xy}$	stresses at a point
σ_x^0, σ_y^0 and σ_{xy}^0	in-plane stresses due to external load
$\tau_{xy}, \tau_{xz}, \tau_{yz}$	shear stresses in xy, xz and yz planes respectively
Φ	angle of twist of the twisted panel
$\partial/\partial x$,$\partial/\partial y$	Partial derivatives with respect to x and y

CHAPTER 1

INTRODUCTION

1. INTRODUCTION

1.1 Introduction

Laminated composite plates have increasing applications due to their high stiffness and strength-to-weight ratios, high fatigue life, resistance to corrosion and other properties of composites. A true understanding is required about the vibration and buckling behaviour of such structures such as the buckling loads and modal characteristics, the distributions of stresses and strains, and the large deflection behaviour. A large amount of research has been devoted to the analysis of vibration, buckling and post buckling behaviour, failure and so on of such structures.

The laminated composite panels are mainly used in shipbuilding, aerospace and in engineering constructions as well. These structures are highly sensitive to geometrical and mechanical imperfections. The defects include different directions of fibre, variations in thickness, delaminations or initial deformations. Plates in a ship structure are subjected to any combination of in plane, out of plane and shear loads. Due to the geometry and nature of loading of the ship hull, buckling is one of the most important failure criteria of these structures.

The twisted cantilever panels have significant applications in turbine blades, compressor blades, fan blades, aircraft or marine propellers, chopper blades, and predominantly in gas turbines. Today twisted plates are key structural units in the research field. Because of the use of twisted plates in turbo-machinery, aeronautical and aerospace industries and so on, it is mandatory to understand both the buckling and vibration characteristics of the twisted plates. The twisted plates are also subjected to loads due to fluid pressure or transverse loads.

1.2 Importance of the present Study

The blades are often subjected to axial periodic forces due to axial components of aerodynamic or hydrodynamic forces acting on it. Composite materials have been increasingly used in turbo-machinery blades for their high specific strength and stiffness. These can be designed through the variation of fibre orientation and stacking sequence to obtain an efficient design**.** For a complete buckling study, a geometrically nonlinear analysis should be carried out. Non-linearity due to material and boundary conditions can also be investigated if required. In a geometrically nonlinear analysis, the stiffness matrix of the structure is updated between loading increments to take into account deformations which affect the structural behaviour. A nonlinear buckling analysis can be performed on the original structure either without imperfection, or by incorporating an imperfection based upon a deformed shape obtained from a linear buckling analysis.

Material nonlinearity during buckling is due to yielding or boundary nonlinearity. Modelling of nonlinear effects should be done in such a way so as to assess the results of additional modelling at every stage. This helps to understand the structural behaviour. A nonlinear analysis calculates actual displacements and stresses as opposed to linear buckling analysis, which only calculates the potential buckling shape.

ANSYS is the software used for analysing and validating the results. Shell element eight node 281 is used which is having 6 degrees of freedom per node with different sizes of mesh and also shell 93 which is having 6 degrees of freedom per node was used. ANSYS is also a powerful tool for non linear analysis.

1.3 OUTLINE OF PRESENT WORK

An attempt has been made to investigate the buckling characteristics of square and rectangular laminated composite twisted plates by a non-linear analysis. The effect of boundary conditions, angle of twist, aspect ratio, number of layers and fiber orientation on the non-linear buckling load is studied. All the studies have been done with the finite element package ANSYS 13.0.

This thesis consists of five chapters. The first chapter gives a brief introduction about the importance of the present study.

In the second chapter, the relevant literature is presented. All the research papers relevant to this study was critically reviewed and discussed briefly.

The third chapter presents the theoretical formulation. All the computation was carried out using ANSYS. This chapter presents the details of modelling in ANSYS. The relevant steps are clearly discussed.

In chapter 4, the results obtained in the present investigation are studied. The effects of various parameters – boundary conditions, number of layers, angle of twist, aspect ratio, fiber orientation, etc is studied and analysed. Chapter 5 gives the conclusions and scope of future work.

CHAPTER 2

LITERATURE REVIEW

2. LITERATURE REVIEW

2.1. LITERATURE REVIEW

The vast use of turbo machinery blades lead to large amount of research over the years. Due to its broad range of application in the practical field, it is essential to know the deformation, vibration and stability behavior of cantilever twisted plates. Due to their widespread use, the mechanical performance of cantilever beams and plates has been studied broadly. One important aspect that has received comparatively little attention involves the coupling of in-plane loads to the out-of-plane deflection of cantilever plates. Such type of mechanism can potentially lead to buckling, and is thus of particular relevance to the design of instrumentation and structures employing these devices.

Louis Bauer and Edward L. Reiss [15] studied the nonlinear deflections of a thin elastic simply-supported rectangular plate. The plate considered was loaded by a compressive force applied along the short edges. They proved that the plate cannot buckle for thrusts less than or equal to the lowest eigenvalue of the linearized buckling problem. For larger loads, fairly accurate solutions of the von Kármán equations were obtained by an accelerated iteration method.

Crispino and Benson [8] studied the stability of thin, rectangular, orthotropic plates which were in a state of tension and twist. Results were presented, for a range of material, geometric and loading parameters, in a compact non-dimensional form.

A computational model for buckling and post buckling analysis of stiffened panels was developed by Eirik Byklum and Jorgen Amdhal [10] which provided accurate results for use in design of ships and offshore structures. The in plane compression or tension, shear force and lateral pressures were considered. The geometric and material non linearity were taken into account, since the onset of yielding was taken as the limit. Various computations were performed for verification of the proposed model and comparisons were made with non linear finite element methods**.** The most important advantage of the method was a large gain in computational efficiency and additionally no geometric modelling was required and no mesh was created.

Nonlinear buckling analysis of shear deformable plates was studied by Judha Purbolaksono and M. H. (Ferri) Aliabadi [14]. Two models of imperfections introduced are: small uniform transverse loads and distributed transverse loads, according to the number of half-waves indicated by the eigenvectors from linear elastic buckling analysis. A simple numerical algorithm was given to analyze the problems. Numerical examples with different loading, geometries and boundary conditions were presented to demonstrate the accuracy of the formulation.

Shaikh Akhlaque-E-Rasul and Rajamohan Ganesan [21] developed a simplified methodology to predict the stability limit load that required only two load steps. A large number of load steps were necessary to determine the buckling load based on the non-linear analysis in which the stability limit load was calculated from the non-linear load–deflection curve. The stability limit loads of the tapered curved plates were calculated. Based on first-ply failure analysis and the simplified non-linear buckling analysis, critical sizes and parameters of the tapered curved plates were determined. The stability limit loads calculated using the present methodology were in good agreement with that calculated based on the non-linear load–deflection curve with the conventional non-linear buckling analysis methodology.

Harvey C. Lee [13] determined the critical buckling pressure of a submarine using Finite Element Analysis (FEA). The problem required to take into account the out-of-roundness of the cylindrical hull due to manufacturing tolerances as well as material nonlinearity. Finite Element Analysis was used to find out the crushing depth of a given submarine design once its buckling strength had been determined.

A.H. Sofiyev et al [2] examined the buckling behaviour of cross-ply laminated non-homogeneous orthotropic truncated conical shells under a uniform axial load. The basic relations of the cross-ply laminated non-homogeneous orthotropic truncated conical shells were derived using the large deformation theory. The influence of the degree of non-homogeneity, the number as well as ordering of layers and the variations of the conical shell characteristics on the non-linear axial buckling load were investigated.

Alinia et al[1] investigated the inelastic buckling behavior of thick plates under interactive shear and in-plane bending. Plate girder web panels and infill plates in steel shear wall systems are two typical elements that are usually subjected to interactive shear and in-plane bending.

Buckling of a cantilever plate uniformly loaded in its plane was studied by Michael J. Lachut and John E. Sader[16]. Applications of this problem consist of loading due to uniform temperature and surface stress changes. This was achieved by a scaling analysis and full three-dimensional numerical solution, leading to explicit formulas for the buckling loads.

The nonlinear buckling and post-buckling behaviour of functionally graded stiffened thin circular cylindrical shells subjected to external pressure were investigated by Dao Van Dung and Le Kha Hoa[6]. The material properties of shell and stiffeners were assumed to be constantly graded in the thickness direction. Fundamental relations and equilibrium equations were derived based on the smeared stiffeners technique and the classical shell theory with the geometrical nonlinearity in von Karman sense. The numerical results show the effectiveness of stiffeners in enhancing the stability of shells.

Dao HuyBich et al [7] presented an analytical approach to investigate the nonlinear static and dynamic buckling of imperfect eccentrically stiffened functionally graded thin circular cylindrical shells subjected to axial compression. Based on the classical thin shell theory with the geometrical nonlinearity in von Karman–Donnell sense, initial geometrical imperfection and the smeared stiffeners technique, the principal equations of motion of eccentrically stiffened functionally graded circular cylindrical shells were derived. The nonlinear dynamic responses were found by using fourth-order Runge–Kutta method. The obtained results show the effects of stiffeners and input factors on the static and dynamic buckling behaviour of these structures.

Z. Yuan and X.Wang [25] studied the non-linear buckling analysis of inclined circular cylinder-in-cylinder by the discrete singular convolution. The non linear differential equation was solved incrementally using the discrete singular convolution (DSC) algorithm mutually with the Newton–Raphson method. It was verified that under certain circumstances only lateral or helical buckling alone will occur. Under some other conditions, both lateral buckling and helical buckling may occur and the critical helical buckling loads were higher than the critical lateral buckling loads if friction was not considered.

M. Shariyat [18] investigated dynamic buckling of imperfect sandwich plates subjected to thermo-mechanical loads. He proposed a generalized higher-order global–local theory that satisfied all the kinematic and transverse stress continuity conditions at the interfaces of the layers.

Danial Panahandeh-Shahraki et al [5] analysed laminated composite cylindrical panels resting on tensionless foundation under axial compression. The governing equations were derived based on classical shell theory and principle of minimum total potential energy. The numerical results showed that if initial curvature of reinforcing panels for modelling columns having cross sections of curved boundaries was neglected, the buckling load would be less than that of actual value. They observed that the influence of tensionless foundation on the uni-lateral buckling behaviour of panels was dependent on parameters such as central angle, aspect ratio, thickness, and the degree of foundation modulus. The effect of parameters like aspect ratio, thickness, central angle, the number and angle of plies, lamination scheme, material orthotropy and foundation modulus on buckling load were also investigated.

Using higher order shear deformation theory, buckling of composite plate assemblies was studied by F.A. Fazzolari et al [11]. A precise dynamic stiffness element based on higher order shear deformation theory was developed for the first time to carry out a buckling analysis of composite plate assemblies. The theory of minimum potential energy was used to derive the governing differential equations and natural boundary conditions. The dynamic stiffness matrix, which includes contributions from the stiffness and initial pre-stress terms, was developed by imposing the geometric boundary conditions in algebraic form. The critical buckling loads and mode shapes of laminated composite plates including stiffened plates was computed using an algorithm proposed by Wittrick–Williams. The effects of important parameters such as thickness-to-length ratio, orthotropic ratio, number of layers, lay-up and stacking sequence along with boundary conditions on the critical buckling loads and mode shapes was investigated.

2.2 Objective and scope of the present study

A review of literature shows that a lot of work has been done on linear buckling analysis of laminated composite plates either by analytical, numerical methods or using software like ANSYS or others. However, very little work exists on the non-linear buckling analysis of laminated composite plates and laminated composite twisted plates. Hence the present study was taken up.

The stiffness of a structure changes due to the change in shape of the structure during its deformation under loads or due to material property changing due to large deformations. If the deformation is small, then it may be assumed that the shape or material property does not change, that is the initial stiffness of the structure does not change with the deformed shape. This is the fundamental assumption in a linear analysis.

A nonlinear analysis is required when the stiffness of the structure changes due to the deformation of the structure. In a nonlinear analysis, the stiffness does not remain same. It has to be changed with changing geometry or material property. If the change in stiffness is only due to change in shape, then the nonlinear behaviour is defined as geometric nonlinearity. If it is due to changing material property, then the nonlinear behaviour is defined as material nonlinearity.

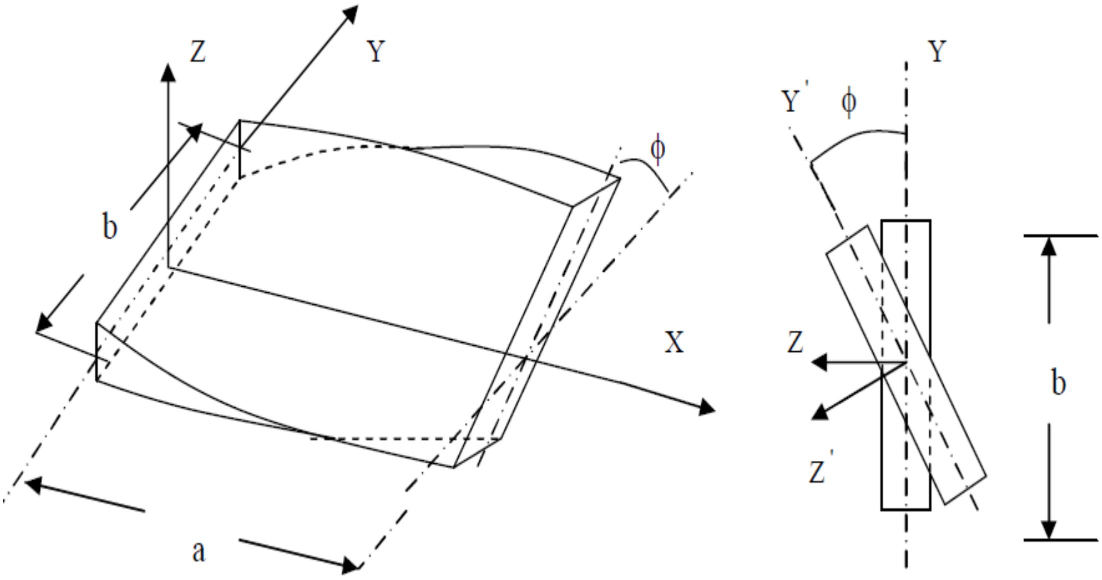
A linear buckling analysis can be used to calculate the Euler buckling load, i.e., the load under which a structure will buckle. Assumptions used in the FEA model may result in the predicted buckling load being much higher for the FEA model than for the actual structure. The results of linear buckling analysis should be used carefully. A nonlinear buckling analysis of a structure thus helps to understand the results in a better way. This is the objective of the present study.

CHAPTER 3

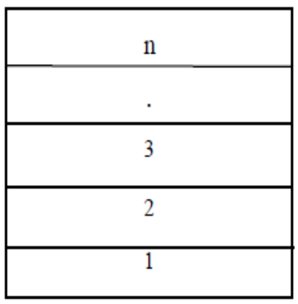
THEORY AND FORMULATION

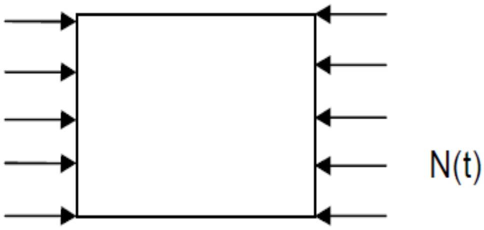
3. THEORY AND FORMULATION

The basic laminated composite twisted curved panel is considered to be composed of composite material laminates. ‘**n**’ denotes the number of layers of the laminated composite twisted panel.



(a) twisted cantilever panel





(b) the lamination **(c) planform subjected to in-plane load**

Figure 3.1: Laminated composite twisted panel with in-plane loads

The governing equations for the laminated composite doubly curved twisted panels/shells subjected to in-plane loading are developed. The governing differential equations have been developed using the first order shear deformation theory (FSDT). The assumptions made in the analysis are given below.

It is assumed that the straight line that is perpendicular to the neutral surface before deformation remains straight but not normal after deformation (FSDT). The thickness of the twisted panel is small compared with the principal radii of curvature. Normal stress in the z-direction is neglected.

3.1 Governing Differential Equations

The governing differential equations are derived on the basis of the principle of potential energy and Lagrange's equation.

The equations of motion are obtained by taking a differential element of the twisted panel as shown in figure 3.2. The figure shows an element with internal forces like membrane forces (N_x, N_y, and N_{xy}), shearing forces (Q_x, and Q_y) and the moment resultants (M_x, M_y and M_{xy}). The governing differential equations of equilibrium of a shear deformable doubly curved pretwisted panel subjected to external in-plane loading can be expressed as (Chandrashekhara[4], Sahu and Datta[19])

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial y} + \frac{Q_x}{R_x} + \frac{Q_y}{R_{xy}} = P_1\frac{\partial^2 u}{\partial t^2} + P_2\frac{\partial^2 \theta_x}{\partial t^2}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{1}{2}\left(\frac{1}{R_y} - \frac{1}{R_x}\right)\frac{\partial M_{xy}}{\partial x} + \frac{Q_y}{R_y} + \frac{Q_x}{R_{xy}} = P_1\frac{\partial^2 v}{\partial t^2} + P_2\frac{\partial^2 \theta_y}{\partial t^2}$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_x}{R_y} - 2\frac{N_{xy}}{R_{xy}} + N_x^0\frac{\partial^2 w}{\partial x^2} + N_y^0\frac{\partial^2 w}{\partial y^2} = P_1\frac{\partial^2 w}{\partial t^2}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = P_1\frac{\partial^2 \theta_x}{\partial t^2} + P_2\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = P_1 \frac{\partial^2 \theta_y}{\partial t^2} + P_2 \frac{\partial^2 v}{\partial t^2} \tag{3.1}$$

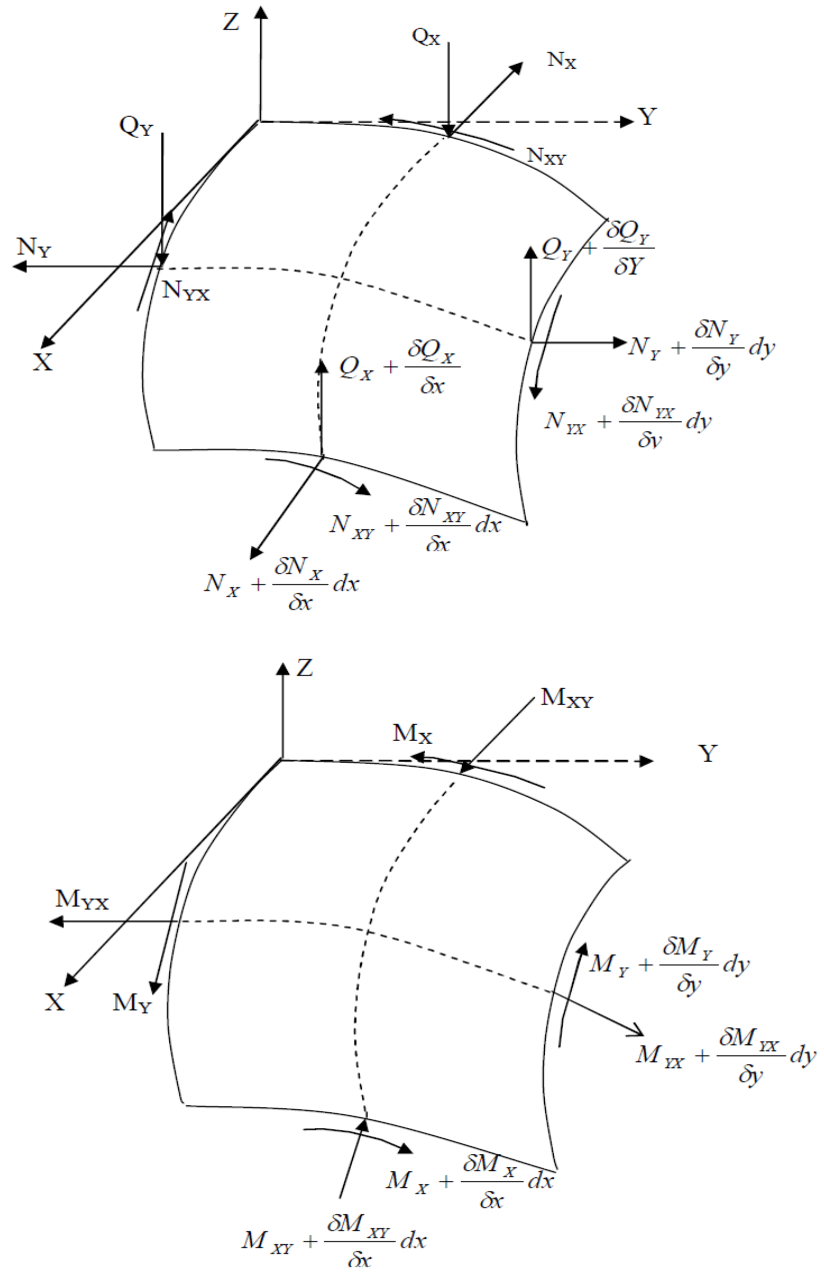


Figure 3.2: Element of a twisted shell panel

Here n = number of layers of panel

$\emptyset$ = angle of twist

a and b are length and width of the panel

Also N_x^0 and Ny^0 are the external loading in the X and Y direction respectively. The constants R_x, R_y and R_{xy} are the radii of curvature in the x and y directions and the radius of twist.

$$(P_1,P_2,P_3) = \sum_{k=1}^{n} \int_{Z_{k-1}}^{Z_k} (\rho)_k (1,z,z^2)dz \tag{3.2}$$

where n= number of layers of the laminated composite twisted curved panel and $(\rho)_k$ = mass density of k_{th} layer from the mid-plane.

First order shear deformation theory is used and the displacement field assumes that the mid –plane normal remains straight before and after deformation, but not necessarily normal after deformation, so that

$$u(x,y,z)= u_0(x,y)+ z\,\theta_y(x,y)$$

$$v(x,y,z)= u_0(x,y)+ z\,\theta_x(x,y) \tag{3.3}$$

$$w(x,y,z)= w_0(x,y)$$

Where u,v,w and u_0, v_0, w_0 are displacement in the x,y,z directions at any point and at the midsurface respectively θx and θ_y and are the rotations of the midsurface normal about the x and y axes respectively .

Strain Displacement Relations

Green-Lagrange’s strain displacement relations are used throughout. The linear strain displacement relations for a twisted shell element are:

$$\xi_{xl}= \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x$$

$$\xi_{yl}= \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y$$

$$\gamma_{xyl}= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy} \tag{3.4}$$

$$\gamma_{xzl}=\frac{\partial w}{\partial x}+\theta_x-\frac{u}{R_x}-\frac{v}{R_{xy}}$$

$$\gamma_{yzl}=\frac{\partial w}{\partial y}+\theta_y-\frac{v}{R_y}-\frac{u}{R_{xy}}$$

Where the bending strains are expressed as

$$k_x=\frac{\partial\theta_x}{\partial x},\ k_y=\frac{\partial\theta_y}{\partial y}$$

$$k_{xy}=\frac{\partial\theta_x}{\partial y}+\frac{\partial\theta_y}{\partial x}+\frac{1}{2}\left(\frac{1}{R_y}-\frac{1}{R_x}\right)\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) \tag{3.5}$$

The linear strains can be expresses in term of displacements as:

$$\{\varepsilon\}=[\mathrm{B}]\{d_e\} \tag{3.6}$$

Where

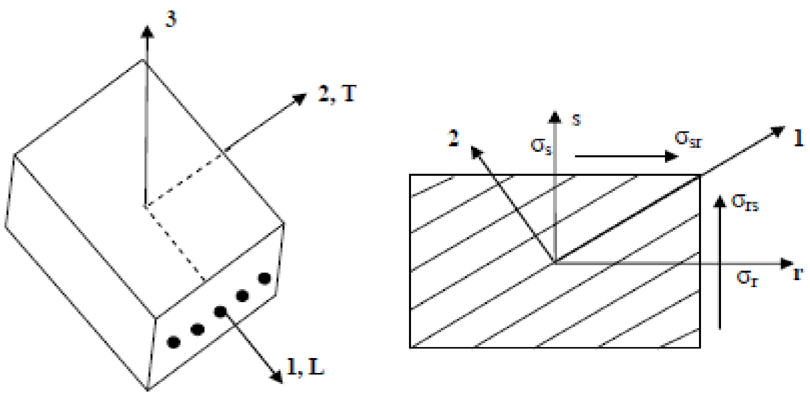
$$\{d_e\}=\{u_1v_1w_1\theta_{x1}\theta_{y1}\ \dots\dots\dots\dots\dots\dots.\,u_8v_8w_8\theta_{x8}\theta_{y8}\} \tag{3.7}$$

$$[\mathrm{B}]=[[B_1]\,,[B_2]..\ \dots\dots\dots\dots\dots....[B_8]] \tag{3.8}$$

$$[B_i]=\begin{bmatrix} N_{i,}x & 0 & \frac{N_i}{R_x} & 0 & 0 \\ 0 & N_{i,}y & \frac{N_i}{R_y} & 0 & 0 \\ N_{i,}y & N_{i,}x & 2\frac{N_i}{R_{xy}} & 0 & 0 \\ 0 & 0 & 0 & N_{i,}x & 0 \\ 0 & 0 & 0 & 0 & N_{i,}y \\ 0 & 0 & 0 & N_{i,}y & N_{i,}x \\ 0 & 0 & N_{i,}x & N_i & 0 \\ 0 & 0 & N_{i,}y & 0 & N_i \end{bmatrix} \tag{3.9}$$

3.2 Constitutive Relations

The basic composite twisted curved panel is considered to be composed of composite material laminates (typically thin layers). The material of each lamina consists of parallel, continuous fibers (e.g. graphite, boron, glass) of one material embedded in a matrix material(e.g. epoxy resin). Each layer may be regarded on a macroscopic scale as being homogeneous and orthotropic. The laminated fiber reinforced shell is assumed to consist of a number of thin laminates as shown in figure 3. The principle material axes are indicated by 1 and 2 and moduli of elasticity of a lamina along these directions are E_{11} and E_{22} respectively. For the plane stress state,$\sigma_{0=0}$



3.3 Laminated shell element showing principal axes and laminate directions

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix} \tag{3.10}$$

Where

$$Q_{11}=\frac{E_{11}}{(1-v_{12}v_{21})} \qquad Q_{12}=\frac{E_{11}v_{21}}{(1-v_{12}v_{21})}$$

$$Q_{21}=\frac{E_{22}}{(1-v_{12}v_{21})} \qquad Q_{22}=\frac{E_{22}}{(1-v_{12}v_{21})} \tag{3.11}$$

$$Q_{66}= G_{12} \qquad Q_{44}= \mathrm{k}G_{13}$$

$$Q_{55}= kG_{23}$$

The on –axis elastic constant matrix corresponding to the fiber direction is given by

$$[Q_{ij}] = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \tag{3.12}$$

If the major and minor Poisson's ratio are ν_{12} and ν_{21}, then using reciprocal relation one obtains the following well known expression

$$\frac{\vartheta_{12}}{E_{11}} = \frac{\vartheta_{21}}{E_{22}} \tag{3.13}$$

Standard coordinate transformation is required to obtain the elastic constant matrix for any arbitrary principle axes with which the material principal axes makes an angle θ . Thus the off-axis elastic constant matrix is obtained from the on-axis elastic constant matrix as

$$[\bar{Q}_{ij}] = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \tag{3.14}$$

$$[\bar{Q}_{ij}] = [T]^T][Q_{ij}][T] \tag{3.15}$$

Where T is transformation matrix. After transformation the elastic stiffness coefficients are:

$$\bar{Q}_{11} = Q_{11}m^4 + 2(Q_{12} + 2Q_{66})m^2n^2 + Q_{22}n^4$$

$$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})m^2n^2 + Q_{12}(m^4 + n^4)$$

$$\bar{Q}_{22} = Q_{11}n^4 + 2(Q_{12} + Q_{66})m^2n^2 + Q_{22}m^4 \tag{3.16}$$

$$\bar{Q}_{16} = (Q_{11} - Q_{12} - 2Q_{66})nm^3 + (Q_{12} - Q_{22} + 2Q_{66})n^3m$$

$$\bar{Q}_{26} = (Q_{11} - Q_{12} - 2Q_{66})mn^3 + (Q_{12} - Q_{22} + 2Q_{66})m^3n$$

$$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})n^2m^2 + Q_{66}(n^4 + m^4)$$

The elastic constant matrix corresponding to transverse shear deformation is

$$\bar{Q}_{44} = G_{13}m^2 + G_{23}n^2$$

$$\bar{Q}_{45} = (G_{13} - G_{23})mn \tag{3.17}$$

$$\bar{Q}_{55} = G_{13}n^2 + G_{23}m^2$$

Where m = cosθ and n = sinθ

The stress strain relations are $\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{xz} \\ \tau_{yz} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} & 0 & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} & 0 & 0 \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \bar{Q}_{44} & \bar{Q}_{45} \\ 0 & 0 & 0 & \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{bmatrix}$ (3.18)

The forces and moment resultants are obtained by integration through the thickness h for stresses as

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_Y \\ M_{xY} \\ Q_x \\ Q_Y \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_x \\ \tau_{xy} \\ \sigma_x z \\ \sigma_{yz} \\ \tau_{xy} z \\ \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{3.19}$$

Where σ_x, σ_y are the normal stresses along X and Y direction τ_{xy},τ_{xz} and τ_{yz} are shear stresses in xy , xz and yz planes respectively.

Considering only in-plane deformation, the constitutive relation for the initial plane stress analysis is

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{21} & A_{22} & A_{26} \\ A_{31} & A_{32} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (3.20)$$

The extensional stiffness for an isotropic material with matrial properties E and v are

$$[D_p] = \begin{bmatrix} \frac{Eh}{1-v^2} & \frac{Eh}{1-v^2} & 0 \\ \frac{vEh}{1-v^2} & \frac{Eh}{1-v^2} & 0 \\ 0 & 0 & \frac{Eh}{2(1+v)} \end{bmatrix} \qquad (3.21)$$

The constitutive relationships for bending transverse shear of a doubly curved shell becomes

$$\begin{Bmatrix} N_x \\ N_p \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{112} & B_{16} & 0 & 0 \\ A_{21} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{44} & S_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ k_x \\ k_y \\ k_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \qquad (3.22)$$

This can also be stated as $\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_j \\ k_j \\ \gamma_m \end{Bmatrix}$ (3.23)

Or $\{F\} = [D]\{\varepsilon\}$ (3.24)

Where A_{ij},B_{ij},$D_{ij}$and S_{ij} are the extensional , bending-stretching coupling, bending and transverse shear stiffness. They may be defined as:

$$A_{ij} = \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^2 - z_{k-1}^2)$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k^3 - z_{k-1}^3)\,;\, i,j = 1,2,6 \tag{3.25}$$

$$S_{ij} = k \sum_{k=1}^{n} \overline{(Q_{ij})}_k (z_k - z_{k-1})\,;\, i,j = 4,5$$

And k is the transverse shear correction factor. The accurate prediction for anisotropic laminates depends on a number of laminate properties and is also problem dependent. A shear correction factor of5/6 is used in the present formulation or all numerical computations.

3.3 ANSYS METHODOLOGY:

For complex geometrical and boundary conditions, analytical method are not so easily adaptable, so numerical methods like finite element method have been used. The finite element formulation is developed here by for the structural analysis of composite twisted shell panels using first order shear deformation theory. ANSYS software which is a finite element software has been used for the study.

The plate is made up with bonded layers, where each lamina is considered to be homogenous and orthotropic and made of unidirectional fiber-reinforced material. The orthotropic axes of symmetry in each lamina are oriented at an arbitrary angle to plate axes. The present study mainly aims to analyse the laminated composite twisted plates under the in plane loading conditions. Methodology involves the linear buckling and non linear buckling analysis of twisted plates. This project consists of developing FEA models of a laminated composite twisted plate under an in plane load. The first step is to develop a model of a laminated composite plate in ANSYS. The model will be subjected to in-plane loads, both linear and non-linear buckling analysis will be studied and results compared with the previous results. Then a laminated composite twisted plate will be studied for its characteristics with in-plane loads. Shell element eight node 281 is used which is having 6 degrees of freedom per node with different sizes of mesh.

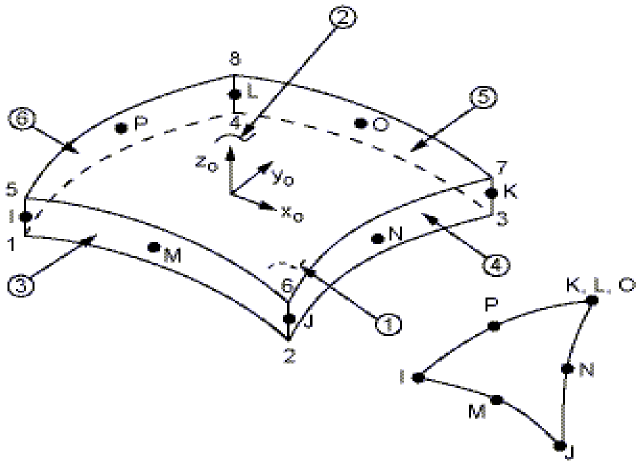


Figure 3.4: A SHELL 281 Element

Modelling and analysis of plate in ANSYS includes the following three steps:

1. Pre- Processor
2. Solution
3. General Post processor.
4. Time History Post Processor

Pre-processing

It is defining the problem

1. Define key points/lines/area/volumes
2. Defining element type and material/geometric properties
3. Creating Mesh lines/area/volumes as required

Solution

Assigning loads, constraints and solving

Here we specify the loads (point or pressure), constraints (translational and rotational), deformations (at the maximum displacement)

Post-processing

Further processing and viewing the results in this stage one may wish to see.

1. List of nodal displacement

2. Element forces and moments

3. Deflection plots

4. Stress contour diagram

5. Load at Failure

Time History Post Processor

Further stage we may wish to see the graphs and behaviour of the material by plotting graphs. Various graphs can be plotted by changing the parameters and the axes we wish to employ and we wish to check. This step is very necessary to view the non linear behaviour as the load can be said from the graph.

The present problem has been solved using ANSYS software. The flat plate was first solved in order to validate the methodology and the results compared to previous results for linear buckling and non linear buckling. Then the methodology was tested for a laminated composite plate with different types of boundary conditions and stacking sequence and results compared to a result from a previous paper. Nonlinear buckling analysis is usually the more accurate approach and is therefore recommended for design or evaluation of actual structures. This technique employs a nonlinear static analysis with gradually increasing loads to seek the load level at which the structure becomes unstable. Using the nonlinear technique, the model can include features such as initial imperfections, plastic behaviour, gaps, and large-deflection response. In addition, using deflection-controlled loading, the post-buckled performance of the structure may be tracked. Nonlinear buckling can be performed on the original structure without imperfection, or by automatically adding an imperfection based upon a scaled deformed shape which could be from a linear buckling model.

CHAPTER 4

RESULTS & DISCUSSION

4. RESULTS AND DISCUSSIONS

In this chapter, the results of linear buckling and non linear buckling analysis of laminated composite plates under in-plane loads is presented. The evaluation is done using ANSYS software.

Non Linear buckling analysis is a static analysis through which we can incorporate the non linearities due to loading, supports and end conditions. Here we consider the geometric non linearity only for our study. After analysing the plate for linear analysis we have to proceed for non linear analysis. We have to give a deformation by applying a small load at the point of maximum displacement obtained from linear buckling analysis. After giving a deformation we will get our analysis done with a geometric non linearity. The load will start decreasing after the solver extracts two number of modes, the load goes on decreasing and then it will increase a little and continue to be constant. The load at which it starts increasing is the buckling load from non linear analysis which is less than the buckling load obtained from linear buckling. A graph is plotted between displacement and load at the node which was given deformation initially. This graph gives the buckling value.

4.1 Convergence study and validation of results

The convergence study is first done for square isotropic plates clamped on all the edges for different mesh divisions and is shown in Table 4.1. Based on this study, a 12 x 12 mesh was chosen for solving the problem.

$a = 100mm$, $b = 100mm$, $h = 1mm$, $E = 210GPa$, $\nu = 0.3$

Table 4.1: Convergence study

Mesh division	Buckling load (kN/m)	Non-dimensional Buckling load
6 x 6	204.1	10.2
8 x 8	200.3	10.015
10 x 10	199.72	9.986
12 x 12	199.65	9.983
Sandeep Sing et al [20]		9.96

Next the formulation is run for laminated composite square plates simply supported on all the edges and the results are compared with that of Fazzolari et al [10] as shown in Table 4.2.

Table 4.2: Comparative study of non-dimensional buckling load for a simply supported laminated composite plate

a =100mm, b = 100mm,

$E_{11} = E_{33}$ = 184 Gpa, E_{22} = 9.2 GPa, $\nu_{12} = \nu_{13}$= 0.25,

$G_{12} = G_{13}$ = 5.52 Gpa , G_{23} = 4.6GPa.

Stacking : $0^0/90^0/90^0/0^0$

E_1/E_2	b/h	**Dimensionless Buckling load Fazzolari et al [11]**	**Dimensionless Buckling load Present Study**	**Dimensionless Non Linear Buckling Load**
10	50	11.209	11.94	11.57
	25	11.41	12.19	11.84
20	50	19.48	20.216	19.76
	25	18.82	19.63	19.17

As can be seen from Table 4.2, the results compare quite well with previous literature and it also can be seen that non dimensional non linear buckling value is quite nearer to the published results. Also a non linear analysis gives a lower estimate of the buckling load than a linear elastic analysis.

4.2 Results and Discussions

Having validated the results for linear buckling load, the formulation was applied to simply supported flat plates with varying parameters like aspect ratio, thickness, and angle of twist, etc.

Table 4.3: Variation of buckling load with aspect ratio for a/h of 250 with different ply lay-ups simply supported on all edges

a/h = 250 ,

E_{11}= 141.0GPa, E_{22} = 9.23GPa,

υ_{12}= 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b	Lay ups	Linear Buckling load, N/m	Non Linear Buckling load ,N/m
1	0/90/0	4998	4900
	0/90/90/0	5211.3	5150
	0/90/0/90/0/90/0/90	5048	4950
2	0/90/0	17800	17500
	0/90/90/0	23815	23000
	0/90/0/90/0/90/0/90	23021	22500
3	0/90/0	42428	41000
	0/90/90/0	53598	52000
	0/90/0/90/0/90/0/90	52242	51000

Table 4.4: Variation of buckling load with aspect ratio for a/h of 150 with different ply lay-ups simply supported on all edges.

a/h = 150 ,

E_{11}= 141.0GPa, E_{22} = 9.23GPa, υ_{12}= 0.313,

G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b	Lay ups	Linear Buckling load, N/m	Non Linear Buckling load, N/m
1	0/90/0	23384	21500
	0/90/90/0	24050	23750
	0/90/0/90/0/90/0/90	23321	22500
2	0/90/0	80731	79000
	0/90/90/0	109560	108700
	0/90/0/90/0/90/0/90	106080	105500
3	0/90/0	330260	328000
	0/90/90/0	246060	245000
	0/90/0/90/0/90/0/90	271410	264000

Table 4.5: Variation of buckling load with aspect ratio for a/h of 100 with different ply lay-ups simply supported on all edges

a/h = 100 ,

E_{11}= 141.0GPa, E_{22} = 9.23GPa, υ_{12}= 0.313,

G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b	Lay ups	Linear Buckling load, N/m	Non Linear Buckling load ,N/m
1	0/90/0	77970	76500
	0/90/90/0	81016	80000
	0/90/0/90/0/90/0/90	78537	76500
2	0/90/0	268858	260000
	0/90/90/0	367000	365500
	0/90/0/90/0/90/0/90	334877	332420
3	0/90/0	637851	632000
	0/90/90/0	821563	815000
	0/90/0/90/0/90/0/90	817963	812500

From the above three tables, table 4.3, 4.4, 4.5 it can be seen that the buckling load increases as the number of layers increases and as the aspect ratio increases, i.e., the buckling load is high for rectangular plates than square plates. Also it can be observed that the symmetrical arrangement of 4 layer ply has more buckling value than the 8 layer asymmetrical ply.

Also in all cases the non-linear buckling load is less than the linear buckling load.

To validate the non-linear buckling formulation, Table 4.6 and Table 4.7 shows the comparison of the buckling loads for laminated composite square plates simply supported on all the edges and cantilever conditions , obtained by both linear and non-linear analysis.

Table 4.6: Comparison of linear and nonlinear buckling loads for a laminated composite plate[0°/90°/90°/0°] with different aspect ratio simply supported on all edges:

a/h = 250 ,
E_{11}= 141.0GPa, E_{22} = 9.23GPa,
υ_{12}= 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b	Buckling load (Linear analysis) N/m	Buckling load (Non Linear analysis) N/m
1	5211.3	5150
2	23815	23000
3	53598	52000

Table 4.7: Comparison of linear and nonlinear buckling loads for a laminated composite cantilever plate [0°/90°/90°/0°] with different aspect ratio

a/h = 250 ,

E_{11}= 141.0GPa, E_{22} = 9.23GPa,

υ_{12}= 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b	Buckling load (Linear analysis) N/m	Buckling load (Non Linear analysis) N/m
1	823.76	786
2	823.28	765
3	823.03	760

From the above tables 4.6 and 4.7 it can be said that as the aspect ratio increases the buckling load increases and then decreases for simply supported plates but whereas it remains almost same for the cantilever plates for the same stacking sequence. The buckling load found using the nonlinear analysis is again found lesser than the linear buckling load.

Table 4.8: Variation of Linear and Non Linear buckling load with angle of twist for a/h of 250 with different ply lay-ups for cantilever boundary conditions:

a/h = 250 ,

E_{11}= 141.0GPa, E_{22} = 9.23GPa,

υ_{12}= 0.313, G_{12} = 5.95GPa, G_{23} = 2.96GPa

Angle of Twist , Φ	No: of layers	Linear Buckling Load N/m	Non Linear Buckling Load N/m
10	$[0^0/90^0]$	198.53	195
	$[0^0/90^0/0^0/90^0]$	402.12	396
	$[0^0/90^0/90^{0/}0^0/0^0/90^0/90^{0/}0^0]$	547.3	535
20	$[0^0/90^0]$	171.40	168.5
	$[0^0/90^0/0^0/90^0]$	346.97	336
	$[0^0/90^0/90^{0/}0^0/0^0/90^0/90^{0/}0^0]$	472.13	465
30	$[0^0/90^0]$	140.03	137.8
	$[0^0/90^0/0^0/90^0]$	282.6	276
	$[0^0/90^0/90^{0/}0^0/0^0/90^0/90^{0/}0^0]$	384.2	378

From the above table it can be said that buckling load decreases for increase in angle of twist and also buckling load increases with number of layers for same angle of twist for anti symmetric ply arrangement. Buckling load is more for symmetric ply arrangement as shown for 8 layer ply in the table.

Table 4.9: Variation of buckling load with aspect ratio for a/h of 250 with different ply lay-ups for a cantilever twisted plate with an angle of twist 10^0

a/h = 250

E_{11}= 141.0GPa, E_{22} = 9.23GPa, υ_{12}= 0.313,

G_{12} = 5.95GPa, G_{23} = 2.96GPa $\Phi = 10^0$

a/b	Lay ups	Linear Buckling load, N/m	Non Linear Buckling load ,N/m
1	$0^0/90^0/0^0$	852.59	840
	$0^0/90^0/90^0/0^0$	402.12	396
	$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	547.30	535
2	$0^0/90^0/0^0$	763.42	751
	$0^0/90^0/90^0/0^0$	360.17	352
	$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	490.19	478
3	$0^0/90^0/0^0$	671.1	653.02
	$0^0/90^0/90^0/0^0$	316.72	306
	$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	431.04	421

Table 4.10: Variation of buckling load with aspect ratio for a/h of 200 with different ply lay-ups for a cantilever twisted plate with an angle of twist 10^0

a/h = 200 ,
E_{11}= 141.0GPa, E_{22} = 9.23GPa, υ_{12}= 0.313,
G_{12} = 5.95GPa, G_{23} = 2.96GPa
$\Phi = 10^0$

a/b	Lay ups	Linear Buckling load, N/m	Non Linear Buckling load ,N/m
1	$0^0/90^0/0^0$	1658.1	1631
	$0^0/90^0/90^0/0^0$	1519.2	1473
	$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	1067.9	1036
2	$0^0/90^0/0^0$	1478.6	1454
	$0^0/90^0/90^0/0^0$	1355	1314
	$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	953.87	935

Table 4.11: Variation of buckling load with aspect ratio for a/h of 150 with different ply lay-ups for a cantilever twisted plate with an angle of twist 10^0

a/h = 150 ,

E_{11}= 141.0GPa, E_{22} = 9.23GPa, υ_{12}= 0.313,

G_{12} = 5.95GPa, G_{23} = 2.96GPa

Φ= 10^0

a/b	Lay ups	Linear Buckling load, N/m	Non Linear Buckling load ,N/m
1	$0^0/90^0/0^0$	3918	3820
	$0^0/90^0/90^0/0^0$	3585.4	3495
	$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	2529.6	2467
2	$0^0/90^0/0^0$	3482.6	3396
	$0^0/90^0/90^0/0^0$	3187.7	3109
	$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	2254.7	2201

From above tables 4.9, 4.10, 4.11 shows that for the buckling value is more for symmetric stacking and less for asymmetric stacking for a square plates. The buckling value decreases for different type of stacking with increase in side/ thickness ratio.

Table 4.12: Variation of buckling load with aspect ratio for a/h of 250 and angle of twist for a cantilever twisted plate

a/h = 250 ,

E_{11}= 141.0GPa, E_{22} = 9.23GPa, υ_{12}= 0.313,

G_{12} = 5.95GPa, G_{23} = 2.96GPa

a/b	Angle of Twist , Φ	Linear Buckling load, N/m	Non Linear Buckling load ,N/m
1	10^0	852.59	840
	20^0	735.39	720
	30^0	597.70	579
2	10^0	763.42	751
	20^0	569.01	560
	30^0	426.88	420
3	10^0	671.11	653.02
	20^0	463.94	455
	30^0	397.2	390

From the table 4.12 it can be said that the buckling value decreases with increase in angle of twist and aspect ratio for the same stacking sequence and side thickness ratio.

Table 4.13: Variation of buckling load with a/h ratio with different ply lay-ups for a cantilever twisted plate with an angle of twist 10^0

a/b = 1 E_{11}= 141.0GPa, E_{22} = 9.23GPa, υ_{12}= 0.313,
G_{12} = 5.95GPa, G_{23} = 2.96GPa $\Phi = 10^0$

Lay ups	Non Linear Buckling load ,N/m		
	a/h= 250	**a/h = 200**	**a/h = 150**
$0^0/90^0/0^0$	840	1631	3820
$0^0/90^0/90^0/0^0$	396	1473	3495
$0^0/90^0/90^0/0^0/0^0/90^0/90^0/0^0$	535	1036	2467

As the a/h ratio decreases, that is thickness of the plate increases, the non-linear buckling load increases for a particular ply orientation as observed from Table 4.13.

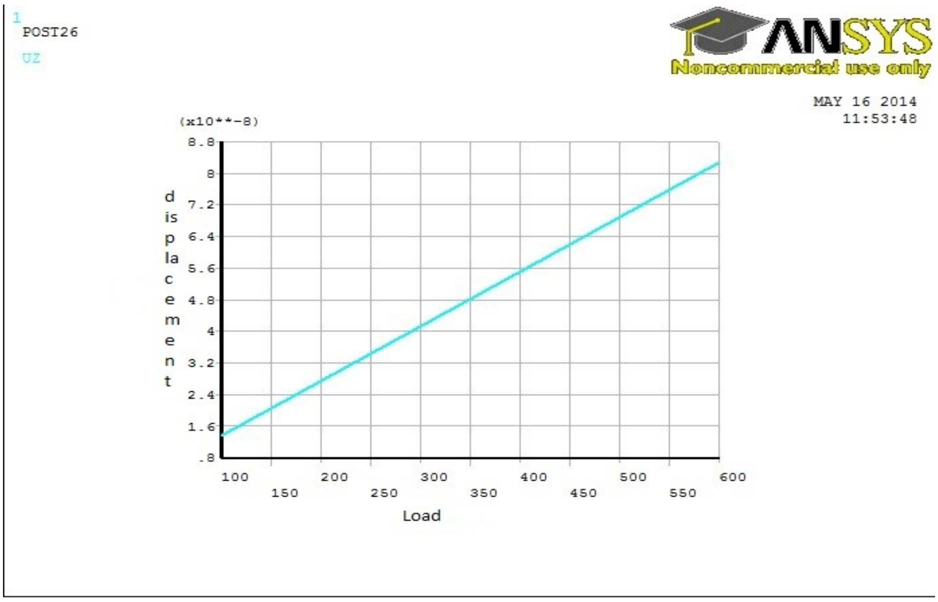
Table 4.14: Variation of buckling load with different Modular ratio for a cantilever twisted plate with an angle of twist 10^0

a/b = 1 E_{22} = 9.23GPa, υ_{12}= 0.313, $\Phi = 10^0$

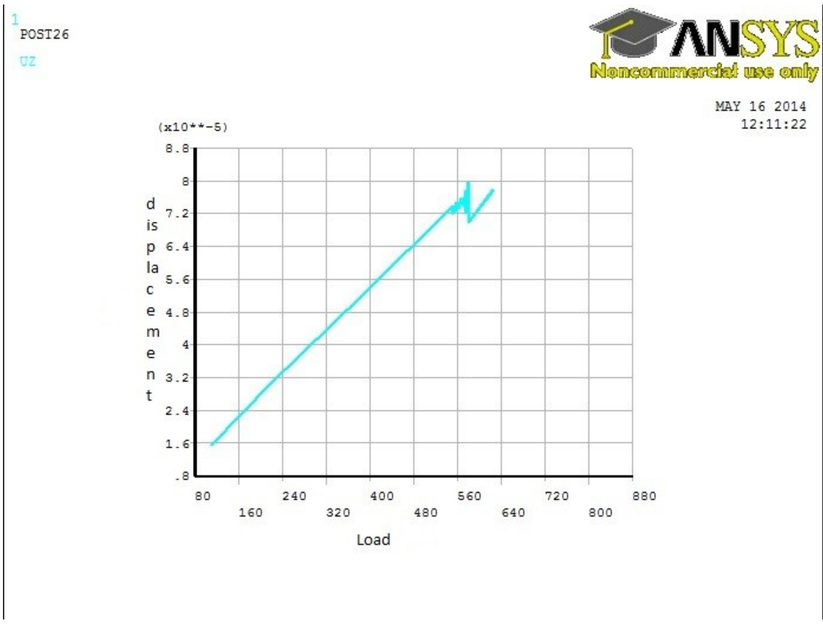
E1/E2	Linear Buckling load, N/m	Non Linear Buckling load ,N/m
10	558.42	546
20	1111.1	1095
30	2214	2160

As the E1/E2 ratio increases, the linear and non-linear buckling load increases for a particular ply orientation as observed from Table 4.14

Figure 4.4: Displacement vs Load for a Cantilever plate for linear and non- linear buckling.



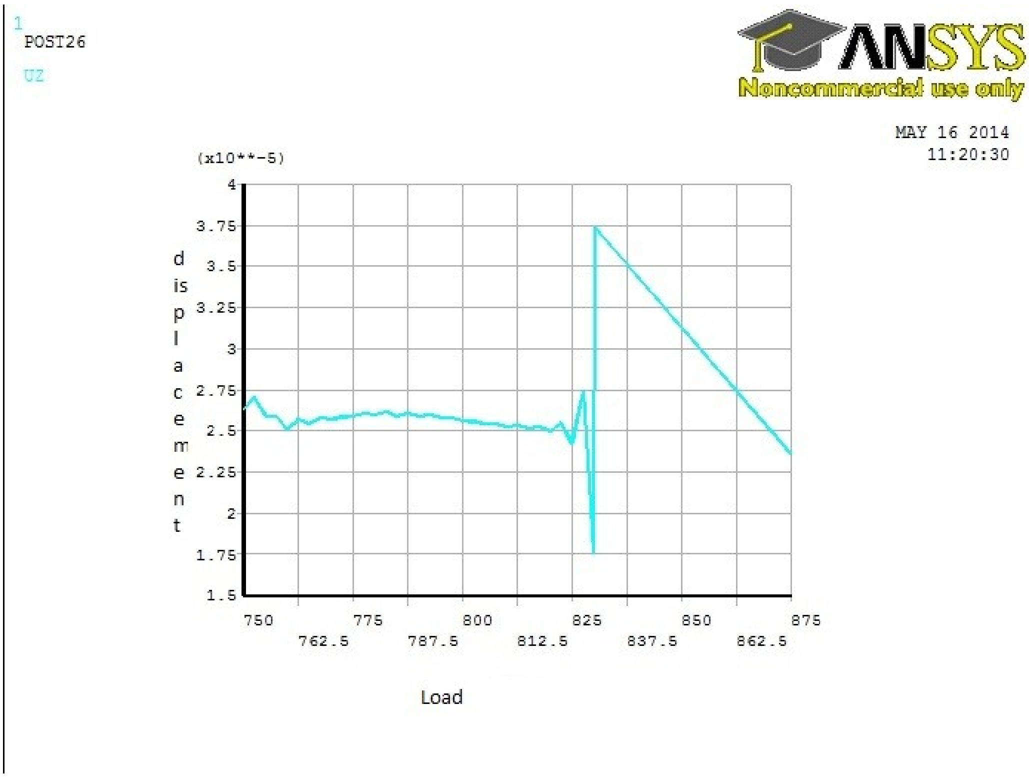
(a). Linear Buckling

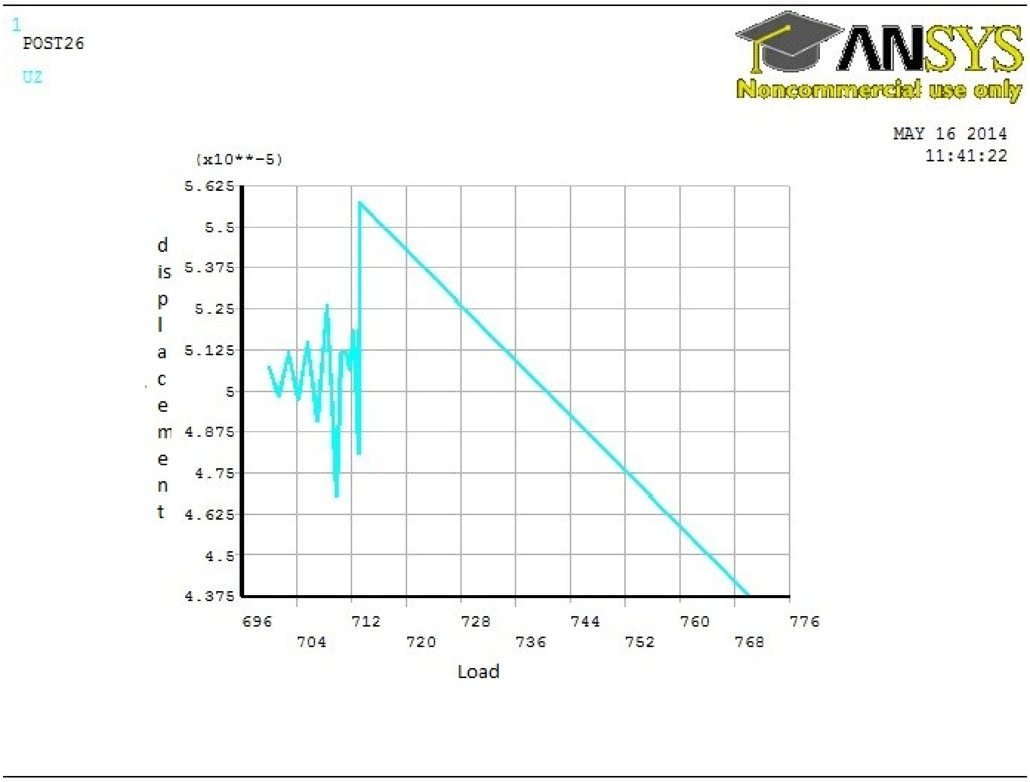


(b). Non Linear Buckling

From the figure 4.6 it can be clearly seen the difference between the linear and non-linear buckling, which shows a sudden jump in displacement with less change in load. The sudden change of displacement with load gives the buckling load of the structure by non-linear analysis.

Figure 4.5: Displacement vs Load for a Cantilever plate with different Angle of Twist for a non-linear analysis:





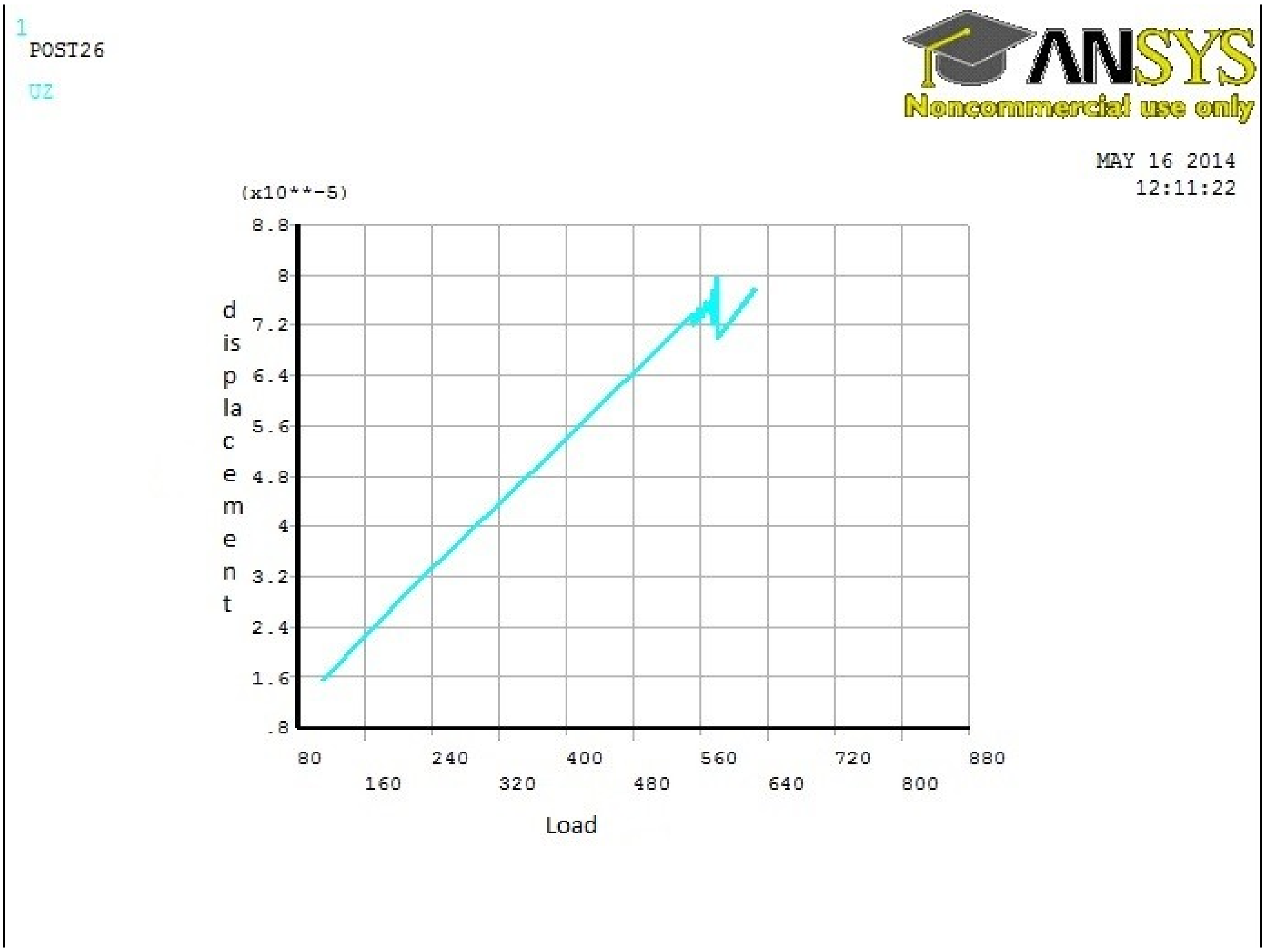


Figure 4.5 shows the various graphs for non linear buckling analysis with different angle of twist which was plotted for Buckling Load Vs Displacement. From the graphs it's clear shown that there is a sudden variation of displacement with a very small increase of load which gives the critical buckling load of the problem by Non linear analysis.

CHAPTER 5

CONCLUSIONS

5. CONCLUSIONS

Instability may occur before a design bifurcation limit is reached. Understanding the large elastic displacement of these types of structures can prevent sudden buckling failures due to applied operational and construction loads.

As discussed earlier, the assumptions made in a linear buckling analysis leads to higher values of the buckling load than is obtained from a nonlinear buckling analysis. This can also be observed from the above studies for both flat and twisted composite plates. Hence the above study validates the necessity of a nonlinear buckling analysis, especially for structures whose shape changes drastically during buckling as is the case for thin shell structures.

From the studies on twisted plates, it is observed that as the aspect ratio increases, the buckling load increases for simply supported plates and decreases for cantilever plates but the non linear buckling load is less than linear buckling load. An increase in the number of layers in general leads to an increase in the buckling load in linear and non linear analysis for the twisted plates. It is also observed that the buckling load increases with decrease in side to thickness ratio for the same aspect ratio for laminated twisted composite plate.

Also it is observed from above studies that as the angle of twist and aspect ratio increases, nonlinear buckling load decreases for laminated composite twisted plates. For the same angle of twist, buckling load increases with number of layers and for symmetric play ups.

As the degree of orthotropy increases for laminated composite twisted plates, the non linear buckling load increases.

The above variations hold true for the non linear studies on both the twisted and flat laminated composite plates.

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